Fuzzy Sets and Fuzzification

Michael J. Watts

http://mike.watts.net.nz

Lecture Outline

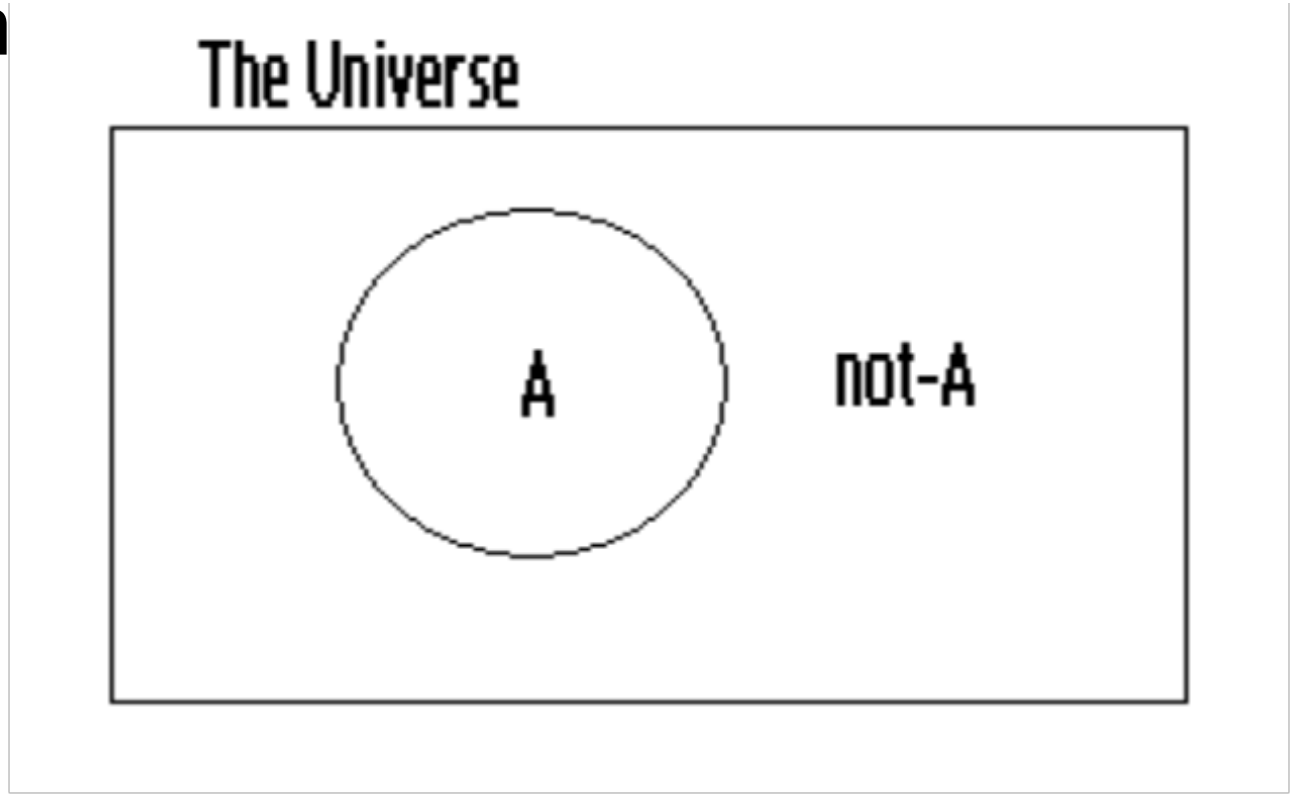
- Crisp sets
- Fuzzy sets
- Fuzzy membership functions
- Fuzzification
- Fuzzy logic

Crisp Sets

- Everything is either true or false
- No uncertainty is allowed
- An item either is
  - entirely within a set, or
  - entirely not in a set
- The Law of the Excluded Middle
  - X must be either in set A or in set not-A
  - no middle ground is allowed

Crisp Sets

- Opposite sets (A and not-A) must between them contain everything
- Venn diagram



Fuzzy Sets

- Items can belong to a fuzzy set to different degrees
  - degrees of membership
- Completely within a set is a membership degree of 1
- Completely outside a set is a membership degree of 0

Fuzzy Sets

- Degrees of membership must sum to 1
- An item can be both A and not-A to different degrees
  - e.g. A to a degree of 0.8, not-A 0.2
- Degrees of membership are expressed with membership functions
- Range of values a variable can take is called the universe of discourse

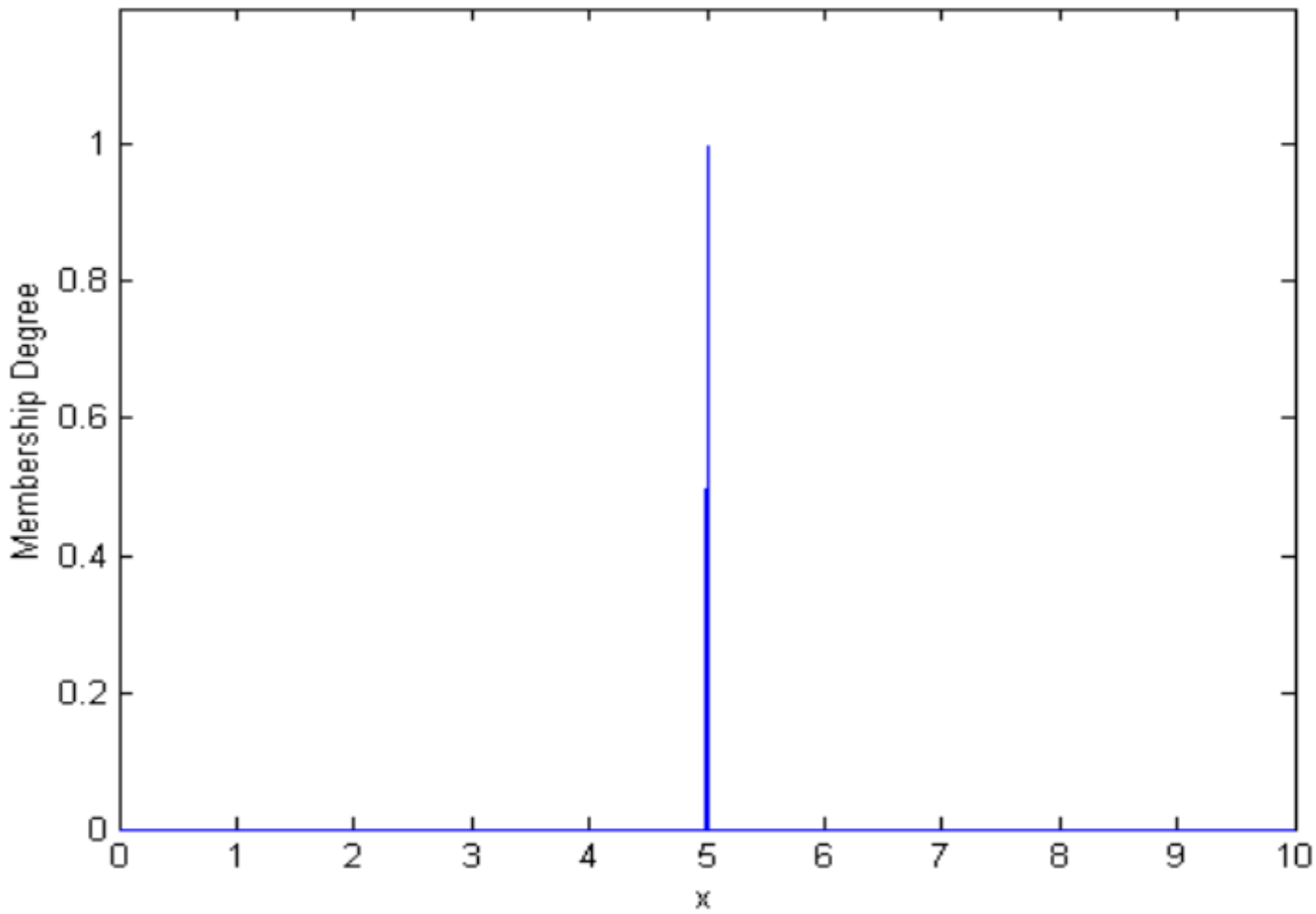
Membership Functions

- A membership function describes the degree of membership of a value in a fuzzy set
- Referred to as MF
  - Also
    - whe $\mu(x)$ the value being fuzzified

Membership Functions

- There are many different types of MF
- Which one to use depends on the problem

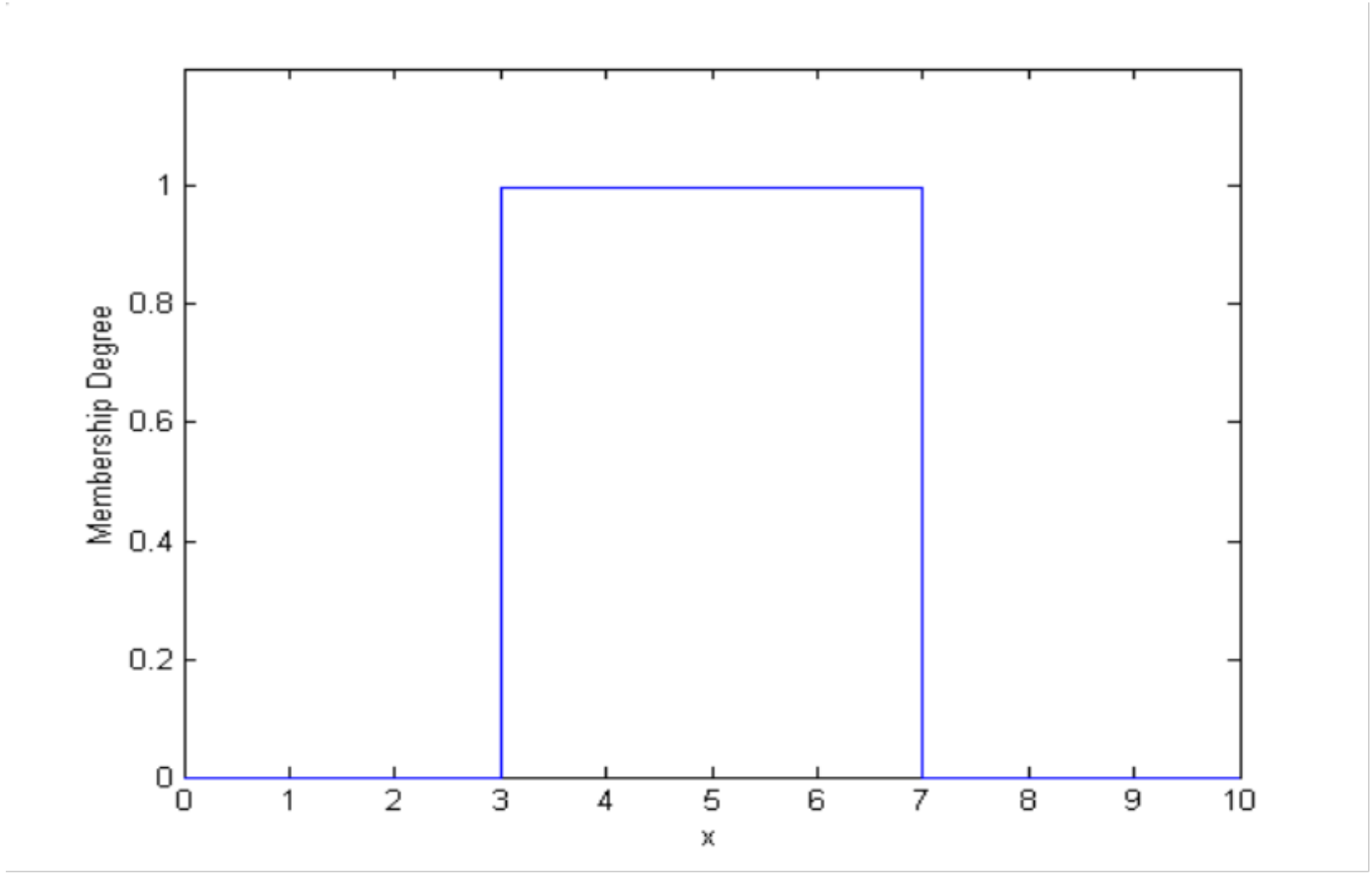
Singleton MF



Singleton MF

$$\mu(x) = \begin{cases} 1, & x = c \\ 0, & \text{otherwise} \end{cases}$$

Rectangular MF



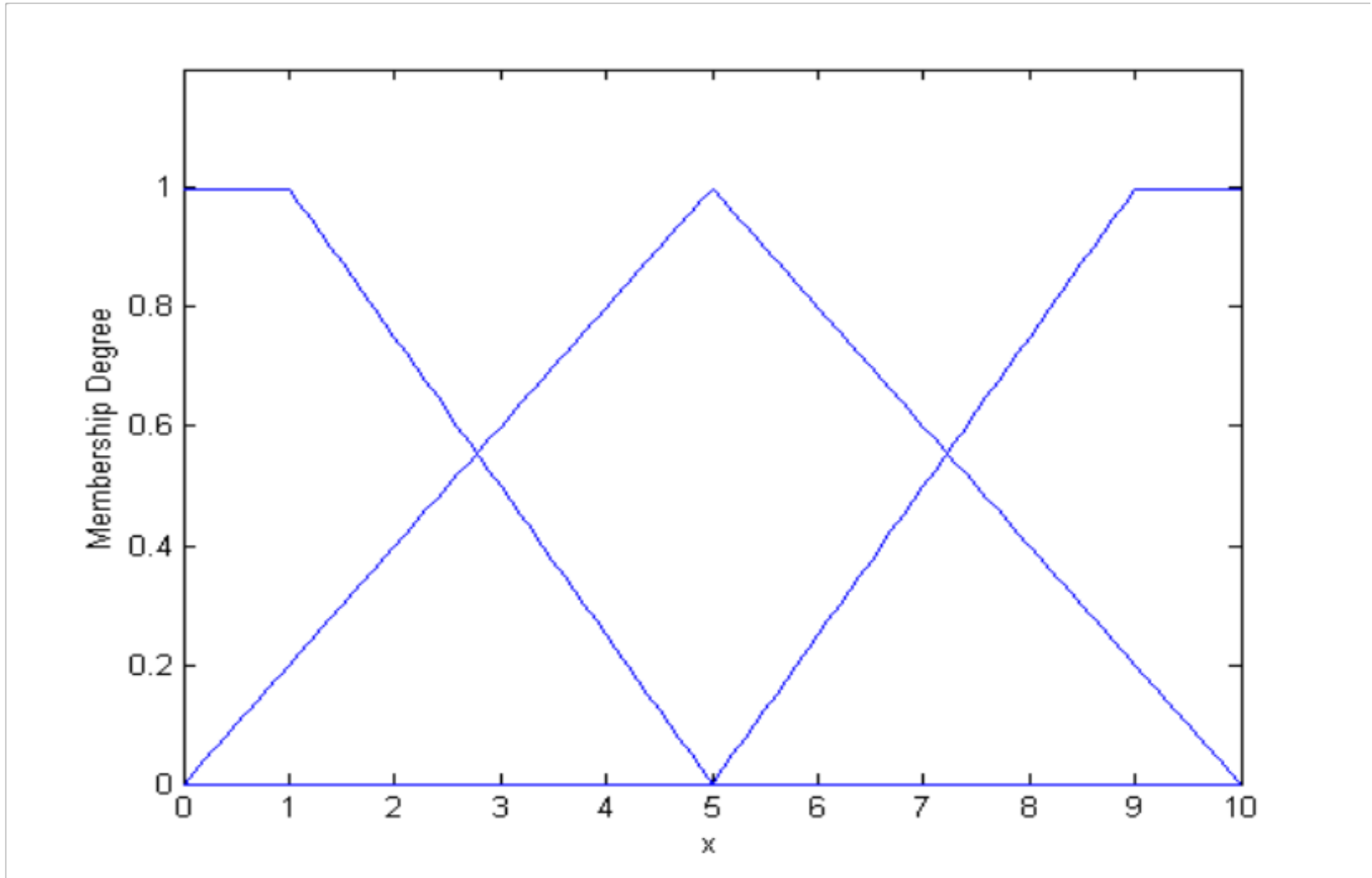
Rectangular MF

$$\mu(x) = \begin{cases} 1, & l \leq x \leq r \\ 0, & \text{otherwise} \end{cases}$$

Triangular MF

- A family of MF
- Constantly tend towards zero and one
- Three in the family
  - Left-shouldered
  - Triangular
  - Right-shouldered

Triangular MF



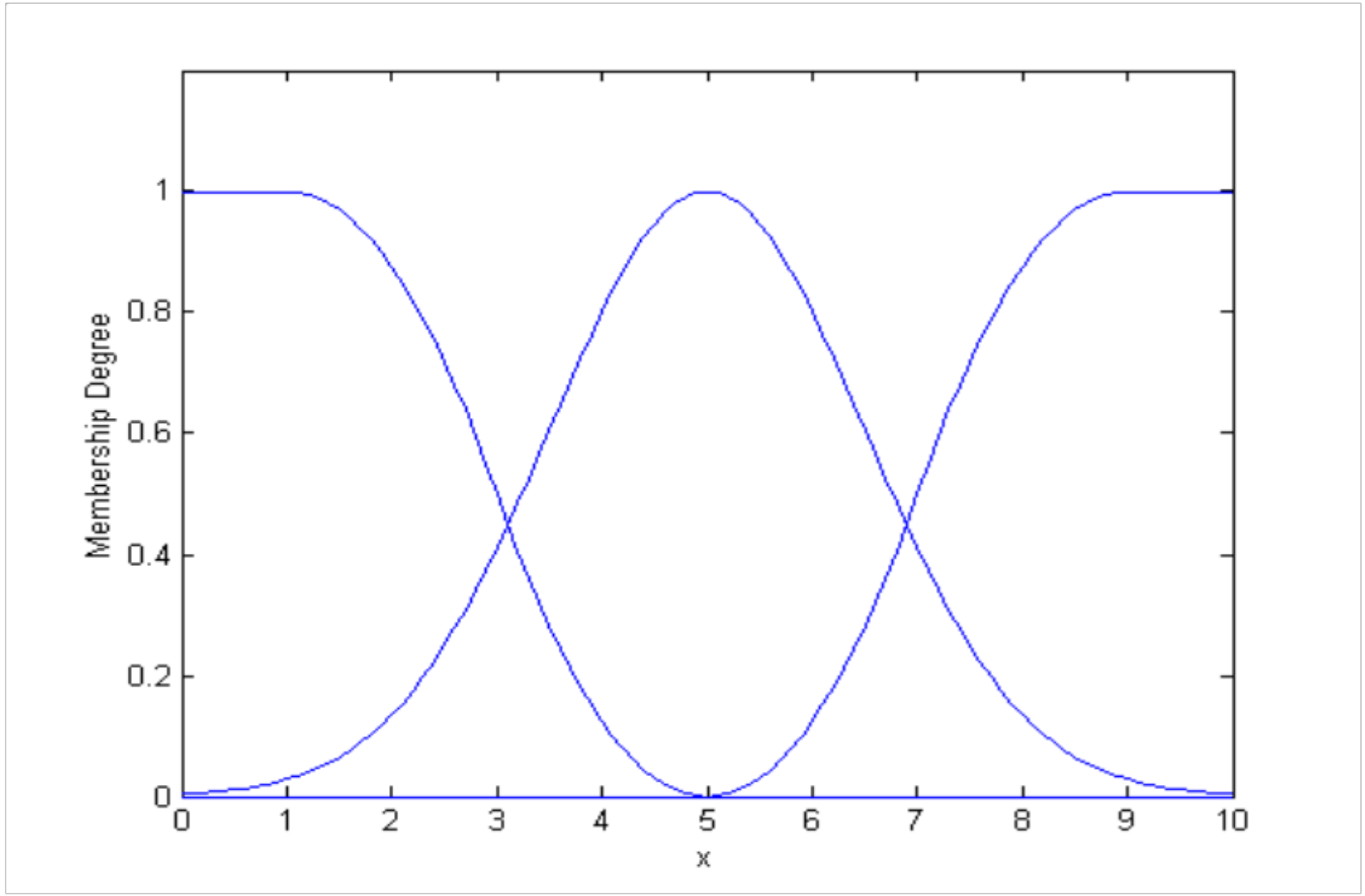
Triangular MF

$$\mu(x) = \begin{cases} 1 - \frac{x-c}{r-c}, & c < x < r \\ 1 - \frac{c-x}{c-l}, & l < x < c \\ 1, & c = x \\ 0, & \texttt{otherwise} \end{cases}$$

Gaussian MF

- A family of MF
- Smoothly tend towards one and zero
- Three in the family
  - *Z*
  - *Gauss*
  - *S*

Gaussian MF



Gaussian MF

$$\mu_b(x) = exp\left(- \frac{(x-c)^2}{2\sigma^2} \right)$$

- *c* is the centre of the MF
- sigma is the width of the MF
- *exp* is the exponential function

Gaussian MF

- *S* function

$$\mu(x) = \begin{cases} 0, & x \leq l \\ 2\left(\frac{x-l}{r-l}\right)^2, & l < x \leq c \\ 1 - 2\left(\frac{x-l}{r-l}\right)^2, & c < x \leq r \\ 1, & x > r \end{cases}$$

Gaussian MF

- *L* is the left hand ‘breakpoint’ of the MF
- *r* is the right hand ‘breakpoint’ of the MF
- *c* is the centre of the MF

Gaussian MF

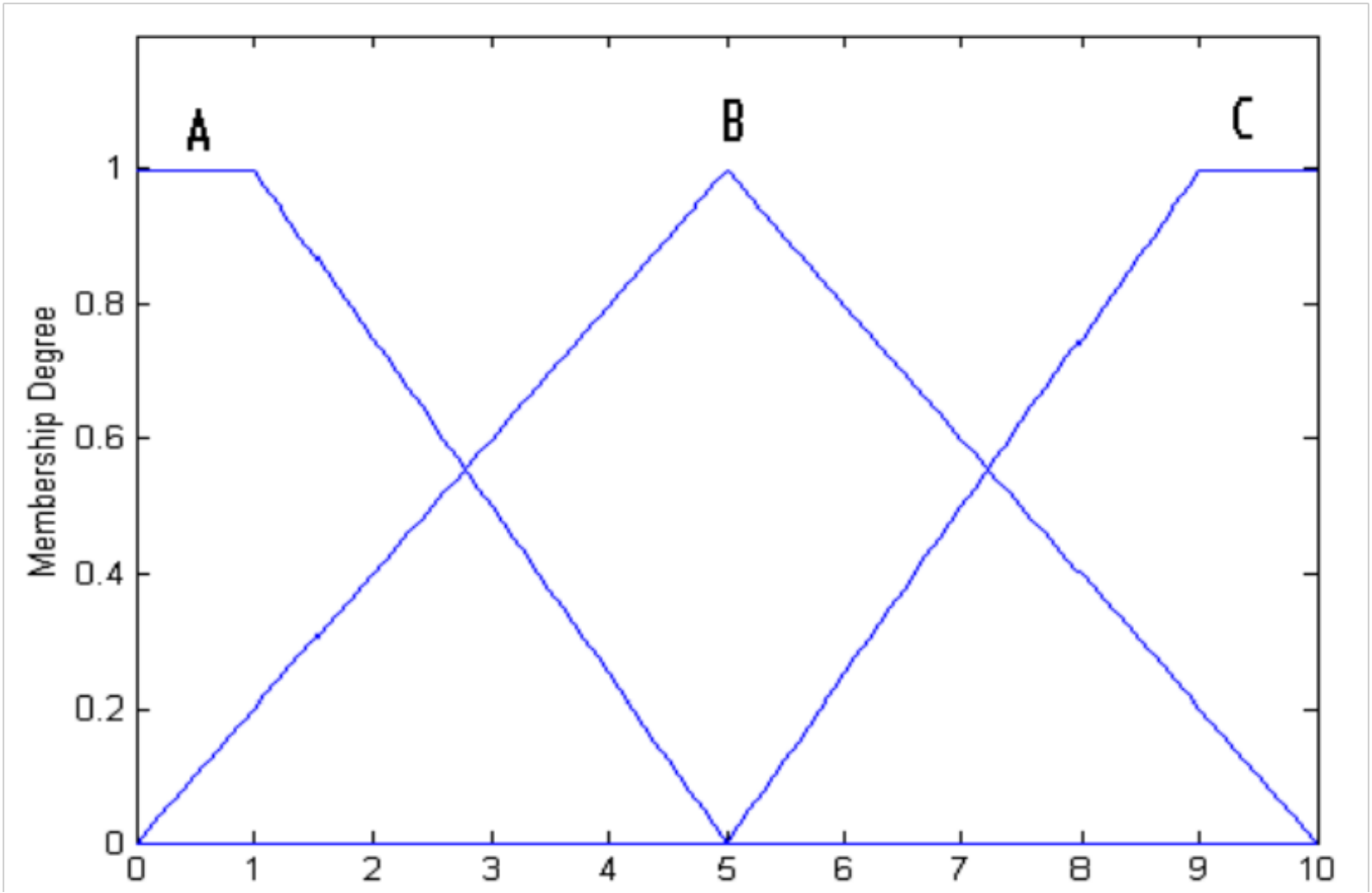
$$\mu(x) = 1 - S(x)$$

- *Z* function is symmetrical to *S* function

Membership Functions

- MF can also be represented by a set of ordered pairs
- Pairs are crisp-fuzzy values
  - A={(0,1.0),(1,1.0),(2,0.75),(3,0.5),(4,0.25),(5,0.0),(6,0.0),(7,0.0),(8,0.0),(9,0.0),(10,0.0)}
  - B={(0,0.0),(1,0.2),(2,0.4),(3,0.6),(4,0.8),(5,1.0),(6,0.8),(7,0.6),(8,0.4),(9,0.2),(10,0.0)}
  - C={(0,0.0),(1,0.0),(2,0.0),(3,0.0),(4,0.0),(5,0.0)(6,0.25),(7,0.5),(8,0.75),(9,1.0),(10,1.0)}

Membership Functions



Fuzzification

- The process of determining the degree to which a value belongs in a fuzzy set
- The value returned by a fuzzy MF
- Most variables in a fuzzy system have multiple MF attached to them
- Fuzzifying that variable involves passing the crisp value through each MF attached to that value

Fuzzy Logic

- Same operations and function as in crisp logic
- Must deal with degrees of truth rather than absolute truths
- Fuzzy logic is a superset of crisp (Boolean) logic

Fuzzy Logic

- AND, OR, NOT
- Crisp logical functions
  - AND true is both parameters are true
  - OR true if either parameter is true
  - NOT reverses truth of argument

Fuzzy Logic

- AND function - crisp version

AND		
A	B	A AND B
0	0	0
0	1	0
1	0	0
1	1	1

Fuzzy Logic

- AND function - fuzzy version
  - take the minimum of the two arguments

AND		
A	B	min(A,B)
0	0	0
0	1	0
1	0	0
1	1	1

Fuzzy Logic

- OR function - crisp version

OR		
A	B	A OR B
0	0	0
0	1	1
1	0	1
1	1	1

Fuzzy Logic

- OR function - fuzzy version
  - take the maximum of the two arguments

OR		
A	B	max(A,C)
0	0	0
0	1	1
1	0	1
1	1	1

Fuzzy Logic

- NOT function - crisp version

A	NOT A
0	1
1	0

Fuzzy Logic

- NOT function - fuzzy version
  - subtract the truth value from one

A	1-A
0	1
1	0

Fuzzy Logic

- Output of fuzzy logical functions are the same as crisp functions
  - just calculated differently
  - handle *degrees* of truth, rather than *absolute* truths
- The basis of fuzzy rule based systems

Summary

- Fuzzy logic deals with uncertainty
- Allows degrees of truth
- Allows partial membership in sets
- Fuzzy membership functions describe degrees of membership in fuzzy sets
- Many different types of MF exist

Summary

- Fuzzification = determining degree of membership
  - uses fuzzy MF to do so
- Fuzzy logic extends Boolean operators to handle partial truths
  - the basis of fuzzy rules